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Economic Applications of Probabilistic Cheap Talk

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Abstract

In this paper we show how Probabilistic Cheap Talk mechanisms (PCT mechanism) can be used to resolve a variety of classical economic problems. We focus on economic applications in public finance and industrial organization. In all but one of the problems we consider, there is a single dominate strategy or nash equilibrium to the one-shot game which is Pareto dominated by other feasible allocations. We show that adding a PCT structure transforms these problems into multi-stage games with a single Pareto dominant, subgame perfect equilibrium. We accomplish this while maintaining the assumptions that moves are made simultaneously, are unobservable to any outside agency, and that payoffs are distributed once and only once.

1. Introduction

In this paper we show how Probabilistic Cheap Talk mechanisms (PCT mechanisms) can be used to resolve a variety of classical economic problems. In a companion paper, Chakravorti, Conley and Taub (1992) we show how an abstract one-shot prisoner's dilemma can be resolved using this type of mechanism. Here, we focus on economic applications in public finance and industrial organization. In all but one of the problems we consider, there is a single dominant strategy or Nash equilibrium to the one-shot game which is Pareto dominated by other feasible allocations. We show that though the use of a PCT mechanism, the equilibrium set can be expanded to include Pareto optimal outcomes. Better still, we show the probabilistic element can be manipulated to generate an equilibrium set with a single Pareto dominant, efficient, individually rational, subgame perfect equilibrium allocation. If fact, in most cases, the equilibrium set can be reduced to consist of only the intuitively appealing Pareto efficient allocation, and the one shot Nash equilibrium allocation. This is accomplished with a mediator who need not be able to observe the strategy choices of the agents, and who knows very little about the economic parameters of the economy. Agents are assumed to know nothing about the other agent's whatsoever.

We retain the basic assumptions of the one-shot PD game in these applications. In particular, we assume that the moves are made simultaneously, and so agents must commit to moves in ignorance of the other agents' moves. Also, we exclude the possibility that an outside agency can impose punishments on agents for failing to behave cooperatively. This means that agents cannot use continent strategies, or sign binding contracts. Finally, the applications we discuss are fundamentally one-shot in nature. Thus, we can't appeal to the repeated games literature for a resolution.

Our objective here is slightly different from the traditional implementation approach. Implementing efficient allocations in a public goods economy is problem with a long history. Jackson and Moulin (1992) give an excellent summery of this literature and propose a very interesting mechanism for implementing a nice class of cost allocation rules. In general, these mechanisms can be quite complicated and unnatural. In addition, there is always a tension between dominant strategy implementation, and the requirements of feasibility, individual rationality, and single valuedness of the equilibrium set. The informational requirements can also vary quite widely. Our approach is to start with what seems to us to be a simple and natural mechanism that can be applied to any one-shot game, and explore the subgame perfect equilibrium allocations. We share with the implementation literature the goal of getting efficient and otherwise desirable allocations as equilibrium outcomes of a mechanism. However, we don't try to implement any decision rule as such. Instead, we show how the PCT mechanism reduces set of equilibrium allocations to the point the problem is reduce to a trivial coordination exercise.

The remainder of the paper is organized as follows. In section 2, we describe a general PCT mechanism. In section 3, we describe a simple public goods problem. We show how the PCT mechanism can be used to generate equal sharing of cost by all agents as the unique Pareto efficient subgame perfect equilibrium. In section 4, we take on the somewhat more difficult public goods revelation problem. We show how agents can be made to truthfully report the benefit they receive from a public good (which is private information), and then share the cost of the good in proportion their report. In section 5, we look at an externality problem. We show how efficient provision of a positive externality, or abatement of a negative externality, can be generated through the use of a PCT mechanism. In section 6, we look at Bertrand oligopoly. We show that all firms setting price at the monopoly level is the unique Pareto optimal subgame perfect equilibrium of a PCT extension of a one-shot game. In section 7, we depart from the mechanism design approach to show how the addition of PCT resolve the chain store paradox. All of these results are achieved using a mediator who knows little about the economic parameters of the game, and who has only the most limited ability to monitor agents' strategy choices. Section 8 concludes.

2. Probabilistic Cheap Talk Mechanisms

In this section we define an abstract PCT mechanism. First, consider a one-shot game $G \equiv < N, M, v >$. Let N be the set of agents. Let M^i be the set of *moves* available to agent $i \in N$. Let $v^i : M^1 \times \ldots \times M^n \equiv M \to \Re^n$, be the payoff function for $i \in N$.

Informally, the PCT extension of the one-shot game is as follows. Agents simultaneously choose moves. These choices are unobservable to any third party (such as a mediator). We restrict attention to this class of games because they cannot be solved by having the agents sign binding contracts. The mediator has a randomization device which he uses to decide between two alternatives: Cheap Talk (CT) and DeadLine (DL). If he chooses CT, the moves made by the agents are payoff irrelevant and are observed by everyone but the mediator. Play then goes to the next round. If he decides on DL then the moves are used to distribute the payoffs from the one-shot game. Play continues until DL is realized. We assume that the mediator is not able to observe the moves sent by the agents at any stage. The only thing he is able to observe is whether all agents make the same move as in the previous round of play (event α) or that at least one agent has revised his move (event β). We can imagine that agents submit their moves in sealed envelopes, and that the mediator can tell when an agent submits a new envelope.

Denote the number of rounds of ex post cheap talk by $t \in \{1, 2 \ldots\}$. The history of talk at t is denoted by h_t. Let $\mathcal{H}$ be the set of all possible histories over all $t \in \{1, 2, \ldots\}$. Let $\mathcal{H}_t$ be the space of all possible histories at time t. We shall set $h_1 = \emptyset$. A *strategy profile* for $i \in N$ is a mapping $s^i = \{s^i_t : \mathcal{H}_t \to M^i\}_{t=0}^{\infty}$. Let S^i be the class of all possible strategy profiles.

If for some $t' < t$ the agents have chosen $m \in M$ in round t', we shall say that the resulting history h_t *contains* m *at* t'. We shall write this as $m \in_{t'} h_t$. A history

$h \in \mathcal{H}$ is said to be *stationary* if

$$\exists\, m \in M \text{ s.t. } \forall\, t \in \{1, 2, \ldots\}, \text{ and } \forall\, t' \leq t, m \in_{t'} h_t.$$

If $h \in \mathcal{H}$ is stationary, then the restriction of h to the first t rounds, h_t, is also said to be stationary. Note that event α generates a stationary history. Let $\mathcal{H}^\alpha$ denote the sub-class of stationary histories generated by event α.

We now define two subclasses of strategies. Let $S_M \in S$ be the class of *stationary trigger strategy profiles* in which agents play a stationary strategy, and respond to any deviation from stationary by going to a "punishment" move the next round:

$$S_M \equiv \{s_{\tilde{m}} \in S\}$$

where for all $h \in \mathcal{H}$, and all $t \in \{1, 2, \ldots\}$

$$s_{\tilde{m},t}(h_t) \equiv \begin{cases} s_{\tilde{m},t-1}(h_{t-1}), & \text{if } \exists\, m \in M \text{ s.t. } m \in_{t-2} h^t, \text{ and } m \in_{t-1} h^t \\ \tilde{m}, & \text{otherwise.} \end{cases}$$

Let $S_M^M \subset S_M$ be the class of *enforcing trigger strategy profiles* in which agents play a *particular* stationary strategy and respond to any deviation from this strategy by going to a punishment move next round. The difference between these two is that in stationary trigger strategies, an agent may or may not require that other agents make a particular move in the first round. However, he always goes to the punishment move if any agent makes a different move in any subsequent round. In an enforcing trigger strategy, punishment is induced not only by nonstationary play, but also by deviation from a specific move by the other players, even in the first round. Formally:

$$S_M^M \equiv \{s_{\tilde{m}}^{\bar{m}} \in S_M\}$$

where for all $h \in \mathcal{H}$, and all $t \in \{1, 2, \ldots\}$.

$$s_{\tilde{m},t}^{\bar{m}}(h_t) = \begin{cases} \bar{m}, & \text{if } \bar{m} \in_{t-1} h_t; \\ \tilde{m}, & \text{otherwise.} \end{cases}$$

Our convention is to have the superscripted move (if any) to be the proposed stationary one, while the subscripted move (if any) is the punishment move. Although our attention will focus on enforcing stationary trigger strategies, the former class is need for technical reasons in the proofs of the lemmata that follow.

For any $i \in N$, and any $m \in M$ let $m^{i,opt}$ denote the set of *optimal one-shot defection from m for agent i.*

$$m^{i,opt} \equiv \{\hat{m}^1 \in M^i \mid \forall \bar{m}^i \in M^i, v^i(m^1, \ldots, \hat{m}^i, \ldots, m^n) \geq v^i(m^i, \ldots, \bar{m}^i, \ldots, m^n)\}.$$

In the applications given below, this will always be single valued. In the general case, it is sufficient to take any element of this set to prove the lemmata below. Note that if $\tilde{m}$ is a Nash equilibrium, then $\tilde{m}^{i,opt} = \tilde{m}^i$.

We are now able to give a formal definition. A PCT mechanism is a profile $\delta = \{\delta_t : h_t \to [0,1]\}_{t=0}^{\infty}$ such that for all $t \in \{1, 2, \ldots\}$,

$$\delta_t(h_t) = \begin{cases} \delta & \text{if } h_{t-1} \in \mathcal{H}^\alpha \\ 1 & \text{if } h_{t-1} \notin \mathcal{H}^\alpha. \end{cases}$$

Given a history h_t, $\delta_t(h_t)$ is the probability that DL is realized at round t. Thus, if the game continues to round t, and the history has been stationary up until then (that is, up to $t-1$,) then the game ends with probability δ. However, the game ends with certainty, and the payoffs are distributed, on the first round after any non-stationary play is seen.[1] Let $\Gamma(G, \delta)$ denote the multi-stage game induced by the PCT mechanism, δ, given the underlying one-shot game, G

For any agent $i \in N$, participating in a strategy profile $s \in S$ yields the following expected payoff at time t, given history h:

$$\delta_t(h_t) v^i(s_t(h_t))$$

[1] We use δ to mean both the profile of maps from histories into probabilities, and the actual probability with which the game ends if the history is stationary. This is a slight abuse of notation, but it should not cause any confusion in context. Also note that we immediately restrict attention to the class of profiles that enforce stationary. We call this class D_2 in a companion paper Chakravorti, Conley and Taub (1992)

$$+(1-\delta_t(h_t))\delta_{t+1}(h_{t+1})v^i(s_{t+1}(h_{t+1}))$$

$$+(1-\delta_t(h_t))(1-\delta_{t+1}(h_{t+1}))\delta_{t+2}(h_{t+2})v^i(s_{t+2}(h_{t+2}))+\ldots$$

Therefore, define the *expected value of strategy s to agent i at time t*, $E_t^i : S \times \mathcal{H}_t \rightarrow \Re_+$ as

$$E_t^i(s,h_t) \equiv \left\{ \delta_t(h_t)v^i(s_t(h_t)) + \sum_{k=t+1}^{\infty} \left[\delta_k(h_k)v^i(s_k(h_k)) \prod_{r=t}^{k-1}(1-\delta_r(h_r)) \right] \right\}$$

where the histories after t are generated by equilibrium play of s given h_t. A strategy profile $s \in S$ is *subgame perfect equilibrium (SPE) of the game* $\Gamma(G,\delta)$ if

$$\forall\, t \in \{1,2,\ldots\},\ \forall\, i \in N,\ \forall\, \bar{s}^i \in S^i, \text{ and } \forall\, h \in \mathcal{H},$$

$$E_t^i(s,h_t) \geq E_t^i(s^1,\ldots,\bar{s}^i \ldots, s^n, h_t)).$$

The following lemmata will be useful in proving the results in subsequent sections. The first of these simply says that if a strategy profile is an SPE of a game, then it must a stationary trigger strategy. Note that this means that the only equilibrium histories are stationary histories.

Lemma 1. *Let $\tilde{m} \in M$ be the unique Nash equilibrium to a one-shot game G. If a strategy, s, is an SPE of $\Gamma(G,\delta)$, then there is $s_{\tilde{m}} \in S_M$ such that $s \equiv s_{\tilde{m}}$.*

Proof/

First, in any SPE strategy, it must be the case that $\tilde{m}$ is played on the round after any nonstationary move occurs. This is because by construction of the PCT mechanism, the game ends with certainty in the next round, and by hypothesis, $\tilde{m}$ is the only Nash equilibrium in such a subgame.

Second, in any SPE, all agents must make the same move each round if the history has been stationary. Suppose instead it was not optimal to play a stationary strategy. Then suppose that the history h_{t+2} is stationary up to t, with agents playing $\bar{m} \in M$ each round. However at round $t+1$ the equilibrium move is $\hat{m} \neq \bar{m}$. The expected payoff to agent j from abiding by the "equilibrium" strategy in this subgame is:

$$\delta v^j(\hat{m}) + (1-\delta)v^j(\tilde{m}).$$

This is because if the PCT game continues until round t, there is a probability of δ that the game ends in round $t+1$. If the game does not end in round $t+1$, the mediator ends the game with certainty in $t+2$ due to the nonstationary play. Thus the probability that the game ends at $t+2$ is $1-\delta$. Assume that $\hat{m} \neq \tilde{m}$. Then the expected payoff to agent j from deviating optimally in round $t+1$ is

$$\delta v^j(\hat{m}^1, \ldots \hat{m}^{j,opt}, \ldots, \hat{m}^n) + (1-\delta)v^j(\tilde{m}).$$

But since $\tilde{m}$ is the only one-shot Nash equilibrium, this defection is superior, and the strategy could not have been an SPE.

The argument is similar if it happens that $\hat{m} = \tilde{m}$. In this case, agent j can improve his expected payoff by deviating in round t. Since the he receives the Nash payoffs in $t+1$ anyway, there is no incentive not to defect in the previous round t, and so s could not have been an SPE.

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The next lemma shows that for any enforcing trigger strategy, $s^{\bar{m}}_{\tilde{m}}$ it is a best response for agents to invoke the punishment move, $\tilde{m}$, if there is any deviation the move $\bar{m}$. Note that this lemma does not say that $\bar{m}^i$ is a best response to $\bar{m}^{-i}$.[2]

Lemma 2. *Suppose that $\tilde{m} \in M$ is the only Nash equilibrium of a game G and consider any enforcing trigger strategy $s^{\bar{m}}_{\tilde{m}}$. For any $t \in \{1, 2, \ldots\}$, suppose also that $h_{t+1} \in \mathcal{H}_{t+1}$ is such that for all $t' < t$ $\bar{m} \in_{t'} h_{t+1}$, but $\bar{m} \neq \hat{m} \in_t \mathcal{H}_{t+1}$. Then for*

[2] We follow the convention that $m^{-i} \equiv \{m^1, \ldots, m^{i-1}, m^{i+1}, \ldots m^n\}$

all $i \in N$, is a best response in this subgame to abide by the trigger strategy and play $\tilde{m}^i$ in round $t+1$ and all future rounds.

Proof/

For $t = 1$, under the hypothesis, $h_2 = \{\hat{m}\}$ where for at least one agent j, $\hat{m}^j \neq \tilde{m}^j$. Note that this is history is not consistent with the equilibrium play of the trigger strategy, but is trivially stationary. However, under the trigger strategy, all agents other than j respond to this history by playing $\tilde{m}^{-j}$ in the second round. If the deadline happens not to hit, and the game does not end in the second round, then it certainly ends in the third due to the non-stationary play. Then clearly it is a best response for all agents $i \in N$ to abide by this trigger strategy and play $\tilde{m}^i$ in the second and third rounds.

Finally for any $t > 2$, suppose h_t satisfies the hypothesis. Then the history is stationary at $\tilde{m}$ up until $t-2$, but at $t-1$ at least one agent j, makes the move $\hat{m}^j \neq \tilde{m}^j$. Again, if the deadline happens not to hit in round $t-1$, it certainly ends in round t due to the non-stationary play. Then clearly it is a best response all agents $i \in N$ to abide by this trigger strategy and play $\tilde{m}^j$ in this last round.

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The next lemma shows that all equilibrium histories can be generated by enforcing trigger strategies, and so attention can be restricted to this class.

Lemma 3. *Let $\tilde{m} \in M$ be the unique Nash equilibrium to a one-shot game, G. Let $s_{\tilde{m}} \in S_M$ be an SPE. Then $h \in \mathcal{H}^\alpha$, with $\hat{m} \in_1 h$, is a possible equilibrium history generated by this strategy if and only if $s^m_{\tilde{m}} \in S^M_M$ is also a SPE.*

Proof/

It is immediate that $h \in \mathcal{H}^\alpha$ with $\hat{m} \in_1 h$ is a possible equilibrium history generated by some strategy $s_{\tilde{m}} \in S_M$ if $s^m_{\tilde{m}} \in S^M_M$ is also a SPE since $s^m_{\tilde{m}} \in S_M$.

To see the other half of the implication, suppose that $h \in \mathcal{H}^\alpha$, with $\hat{m} \in_1 h$, is a possible equilibrium history generated by some strategy $s_{\bar{m}} \in S_M$ but that $s_{\hat{m}}^{m} \in S_M^M$ was not an SPE. Then for some $t \in \{1, 2, \ldots\}$, and some $\bar{h}^t \in \mathcal{H}$, there is an agent $i \in N$ and a strategy s_t^i such that

$$E_t^i(s_{\hat{m},t}^{m,1}, \ldots, s_t^i, \ldots, s_{\hat{m},t}^{m,n}, \bar{h}_t) > E_t^i(s_{\hat{m}}^{m}, \bar{h}_t).$$

Suppose first that for all $t' < t$, $\hat{m} \in_{t'} \bar{h}^t$. But since the future play of both $s_{\hat{m}}^{m}$ and $s_{\bar{m}}$, are the same given this history, it follows that:

$$E_t^i(s_{\hat{m},t}^{m,1}, \ldots, s_t^i, \ldots, s_{\hat{m},t}^{m,n}, \bar{h}_t) = E_t^i(s_{\hat{m},t}^1, \ldots, s_t^i, \ldots, s_{\hat{m},t}^n, h_t)$$

$$> E_t^i(s^m, h_t) = E_t^i(s_{\hat{m}}^m, \bar{h}_t),$$

which contradicts the hypothesis that $s_{\hat{m}}$ is an SPE.

Suppose instead that for some $t' < t, \bar{m} \in_{t'} h^t$, and $\bar{m} \neq \hat{m}$. Without loss of generality, assume that round t' is the first time any move other than $\hat{m}$ is seen. Then by lemma 2, it is a best response for agent i to abide by the trigger strategy and invoke the punishment move in all rounds after t'.

Thus, $s_{\hat{m}}$ is an SPE, then $s_{\hat{m}}^{m}$ also has all the agents playing a best response is every subgame, and is therefore an SPE equilibrium as well.

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3. A Hand Raising Mechanism for the Provision of Discrete Public Goods

Consider the following one-shot game $G^{hr} \equiv < N, M, v >$. Each agent $i \in N$, chooses between two moves: $M^i = \{HO, FR\}$ (Help Out, or Free Ride). Let $\#HO : M \rightarrow \{0, \ldots, n\}$ be a function that gives the number of agents that agree to help out for in any given profile of moves. Then for all $i \in N$, and all $m \in M$,

$$v^i(m) = \begin{cases} B^i - \frac{C}{\#HO(m)} & \text{if } m^i = HO \\ B^i & \text{if } m^i = FR \text{ and } \#HO(m) \neq 0 \\ 0 & \text{if } \#HO(m) = 0. \end{cases}$$

We interpret B^i as the benefit that agent i stands to receive if a public project is undertaken, and C as the cost of the project We assume that for all $i \in N, C > B^i > 0$, and $C < \sum_{i=1}^{n} B^i$. This means that no single agent would be willing to build the project on his own, but the sum of the benefits to all agents exceeds the cost.

This is a very simple model of a mechanism to produce and pay for a discrete level of a public good. To fix the idea, suppose that a group of neighbors is considering building a community playground for the local children. By agreeing to help out with the playground, an agent promises to show up at the proposed site and share in the effort needed to complete the project with the other agents who have agreed to do the same. This seems to be a fairly common mechanism in the real world. For example, when we are asked to volunteer for academic committees, the presumption is that all members will share in the work equally, and continue to work until the task is completed.

The difficulty, of course, is in overcoming the free rider problem. In the one-shot game, in which the moves are simultaneous, it is a dominant strategy to free ride. Also notice that building a playground is fundamentally a one-shot problem. It does not make sense to think about building an infinity of playgrounds. We therefore cannot appeal to the repeated games literature for help.

A common feature of these mechanisms in the real world that there is often a probabilistic element involved. That is, not every project that we are asked to volunteer for really gets off the ground right away. People walk around saying we should build a playground in our neighborhood, and pledging that they will help if the project ever gets started. These pledges are cheap talk that people use as a signaling device to get others to participate. At some point, the coalition comes together and the last set of pledges are called in. In this section we show how this element of random delay, which leads to probabilistic cheap talk, can help coalitions of agents to overcome the free rider problem.

Consider the incentive problem facing any particular agent i when all the other agents play their part of an arbitrary trigger strategy $s_{\tilde{m}}^{\bar{m}} \in S_M^M$. Since universal free riding is the only Nash equilibrium of G^{hr}, by lemma 1 we can assume that the punishment move is $\tilde{m} \equiv (FR, \ldots, FR)$. If $\bar{m}^i = FR$, then i is being asked to play his one-shot dominant strategy. Then it is trivially a best response in every subgame for him to play his part in the trigger strategy equilibrium. On the other hand, if $\bar{m}^i = HO$, he finds that playing his part of this strategy is a best response if and only if the following condition is met:

$$B^i - \frac{C}{\#HO(\bar{m})} \geq \delta B^i.$$

The left hand side of this expression is i's payoff from playing along with the trigger strategy. By so doing the agent i guarantees himself this payoff whenever the deadline happens to fall. The right hand side is his payoff from the optimal deviation, free riding from the first round of the subgame. If the deadline hits in round one (which is a probability δ event), then he gets the full benefit of the public good, B^i, without paying any of the cost of the project. If the game does not end, then all other agents free ride in the remaining rounds. This nonstationary play induces the mediator to end the game with certainty in the third round. Thus, the only possible benefit from defecting from the trigger strategy is free riding in the first round in the event that the deadline happens to hit. Then clearly the agent will abide by the trigger strategy if and only if:

$$\delta \leq 1 - \frac{C}{B' \# HO(\tilde{m})}.$$

First, consider the case of identical agents. Let

$$\delta^* = 1 - \frac{C}{B \mid N \mid}$$

and $\bar{m} \equiv (HO, \ldots, HO)$.

Lemma 4. *Suppose all agents in the game G^{hr} are identical. Then for the PCT extension $\Gamma(G^{hr}, \delta^*)$, $s_{\tilde{m}}^{m}$ and $s_{\tilde{m}}^{\bar{m}}$ are the only SPE.*

<u>Proof</u>/

Clearly, $s_{\tilde{m}}^{\tilde{m}}$ is an SPE for any δ since $\tilde{m}$ is the only Nash equilibrium of the one-shot game. To see that $s_{\tilde{m}}^{\bar{m}}$ is also an SPE, consider any $i \in N$ and any $h \in \mathcal{H}$. This history could have evolved in one of two ways.

Suppose first that the game has not ended at any given t, and suppose that all agents have been playing HO each round. Then by construction of δ^* it is a best response for i to play HO in round t.

Suppose on the other hand that at some $t \in \{1, 2, \ldots\}$, some agent j plays FR. Then by lemma 2 it is a best response for all agents to abide by the trigger strategy and play FR in all remaining rounds.

Finally, suppose there was another trigger strategy $s_{\tilde{m}}^{m}$ that is an SPE. But then $\#HO(\bar{m}) > \#HO(\hat{m})$ and so

$$\delta^* B = B - \frac{C}{\#HO(\bar{m})} > B - \frac{C}{\#HO(\hat{m})}.$$

Thus it would be optimal for every agent to defect from any other trigger strategy.

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Theorem 1. *Suppose all agents in the game G^{hr} are identical. Then for the PCT extension $\Gamma(G^{hr}, \delta^*)$, $\{(B - \frac{C}{|N|}, \ldots, B - \frac{C}{|N|}), (0, \ldots 0)\}$ are the only SPE payoffs.*

<u>Proof</u>/

By lemma 4, $s^{\bar{m}}_{\bar{m}}$ and $s^{\tilde{m}}_{\tilde{m}}$ are the only SPE trigger strategies. Then by lemma 3,

$$\bar{h} \equiv \{h \in \mathcal{H} \mid \forall\, t \in \{1, 2, \ldots\}, \bar{m} \in_t \bar{h}\},$$

and

$$\tilde{h} \equiv \{h \in \mathcal{H} \mid \forall\, t \in \{1, 2, \ldots\}, \tilde{m} \in_t \bar{h}\},$$

are the only SPE histories. Thus

$$\{(B - \frac{C}{|N|}, \ldots, B - \frac{C}{|N|}), (0, \ldots 0)\}$$

are the only SPE payoffs.

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Thus, if all agents derive the same benefit from the project, we can choose δ high enough so that everyone helping out, and everyone trying to free ride (and the project not being undertaken) are the only an SPE outcomes. Then since everybody helping out strongly Pareto dominates everybody free ridding, it should be easy for agents to coordinate on the cooperative equilibrium.

Now consider the case of nonidentical agents? We know that an agent is better off helping out than free riding in a trigger strategy enforcing $\bar{m}$ if:

$$B^i - \frac{C}{\# HO(\bar{m})} \geq \delta B^i.$$

Assume that the agents are ordered so that agents with a lower index place more value on the project. For simplicity, assume that all agents receive different benefits. Thus $B^1 > B^2 > \ldots > B^n$.

For any $J \in \{1, \ldots, N\}$, let m^J denote the move in which the first J agents help out and the rest free ride:

$$m^J \equiv (m^{J,1}, \ldots, m^{J,j}, m^{J,j+1}, \ldots, m^{J,n},) = (HO, \ldots HO, FR, \ldots FR).$$

For any $J \in \{1, \ldots, N\}$ make the following definition:

$$\delta^J = 1 - \frac{C}{B^J \# HO(m^J)}.$$

Finally, define J^* to be:

$$J^* \equiv \{J\{1, \ldots, N\} \mid \forall\, I, \delta^J > \delta^I\}.$$

Note that J^* may not be always be a singleton, but is generically unique. We will assume in the following that J^* is unique.

Lemma 5. *Suppose that J^* is unique. Then for the PCT extension $\Gamma(G^{hr}, \delta^{J^*})$, $s_{\tilde{m}}^{m^{J^*}}$ and $s_{\tilde{m}}^{\tilde{m}}$ are the only SPE.*

Proof/

Clearly, $s_{\tilde{m}}^{\tilde{m}}$ is an SPE for any δ since $\tilde{m}$ is the only Nash equilibrium of the one-shot game.

For all agents $i > j^*$, playing strategy $s_{\tilde{m}}^{m^{J^*,i}}$ is obviously a best response in any subgame since they play FR in each round. So consider any agent $j \leq j^*$ and any $h \in \mathcal{H}$. This history could have evolved in one of two ways.

Suppose the game has not ended at any given t, and suppose all other agents have been playing the move given by m^{J^*} each round. Then by construction of δ^{J^*} it is a best response for j to play HO.

Suppose on the other hand that at some $t \in \{1, 2, \ldots\}$, some agent $j \leq j^*$ plays FR. Then by lemma 2 it is a best response for all agents to abide by the trigger strategy and play FR in all remaining rounds.

Finally, suppose there is another trigger strategy $s_{\bar{m}}^{m^I}$ that was an SPE. To see that this could not be observe that for any $I \neq J$

$$\delta^{J^*} B^i > \delta^I B^i = B^i - \frac{C}{\#HO(m^I)}.$$

Thus it would be optimal for agent i to defect from this strategy. Obviously, for any J, the same argument would hold for any coalition that was the same size as J but which did not include the agents with the highest benefit. In this case, the δ needed to support this as an SPE is even lower since the lowest benefit is lower than B^j, and so defection is even more tempting for this lowest benefiting agent under δ^{J^*}.

•

Economically, what this is saying is that if the benefits are unequal, and drop off rapidly, then for the shortest game (highest δ), only the highest benefiting agents will raise their hands in an SPE. For example, we would predict that if parents receive high benefits from the playground, and non-parents receive only a little, it is likely that only the parents will share in the cost in equilibrium. This seems a agree with everyday experience. But what does it mean for the benefits to be very unequal? Below we give a sufficient bound on the rate of decline of the benefit profile for only equal sharing to be an SPE at the highest possible δ.

Lemma 6. *Suppose that for all $i \in N$, $\frac{|I|+1}{|I|} < \frac{B^i}{B^{i+1}}$, then for the PCT extension $\Gamma(G^{hr}, \delta^N)$, $s_{\bar{m}}^{m^n}$ and $s_{\bar{m}}^{\bar{m}}$ are the only SPE.*

Proof/

In this case $J^* = N$. To see this note that for all $i \in N$:

$$\delta^I = 1 - \frac{C}{B^i \# HO(m^I)} < 1 - \frac{C}{B^{i+1}(\# HO(m^I) + 1)} = \delta^{I+1}.$$

Then apply Lemma 5.

•

4. The Proportional Cost Allocation Problem

Now consider the following one-shot game $G^{pc} \equiv < N, M, v >$ where for all $i \in N$, $M^i \equiv \Re_+$, and for all $i \in N$ and all $m \in M$:

$$v^i(m) = \begin{cases} B^i - C\frac{m^i}{\sum_{j=1}^{n} m^j} & \text{if } m^i > 0 \\ B^i & \text{if } m^i = 0 \text{ and } \exists\, j \in N \text{ s.t. } m^j > 0 \\ 0 & \text{if } \forall\, j \in N, m^j = 0. \end{cases}$$

We interpret B^i as the private benefit that agent i receives if the public project is built, and C as the cost of the project. We assume that for all $i \in N, C > B^i > 0$.

This is a problem of producing and paying for a discrete level of a public good in which the cost sharing rule is proportional rather than equal sharing. We find the proportional sharing rule attractive when all agents tell the truth for several reasons. First, it is consistent with the benefit theory of taxation. This is a traditional notion of fairness is public economics. Second, we will show that this solution is identical to the one proposed by Kalai and Smorodinsky (1975) when we view this situation as a bargaining problem.

In the one-shot game, if an agent admits to valuing the public good, he is charged in proportion to his reported value. This means that it is even possible for agent to be forced to pay more than his true benefit if other agents under-report. Unfortunately, it is a dominate strategy for agents not to reveal the true benefit and report a value of zero instead. Of course if no agent reports a positive value than the project is not built, which is not Pareto optimal.

We show in this section how the PCT mechanism can be used to solve this classic public goods revelation problem. We assume that the mediator knows only the total social value of the project. He knows neither the assignment nor the distribution of the private benefits. Also, as before, the mediator can determine whether or not a history h belongs to $\mathcal{H}^{\alpha}$, but cannot directly observe the moves of the agents.

Let $\tilde{m} = (0, \ldots, 0)$. If the move $\tilde{m}$ is made, the project is not built, and all taxes are zero. This is the only Nash equilibrium of the one-shot game G^{pc}.

Lemma 7. *Suppose* $C \leq \sum_{j=1}^{n} B^j$. *Then for* $\delta = 1 - \frac{C}{\sum_{j=1}^{n} B^j}$, $s_{\tilde{m}}^{\bar{m}}$ *is an SPE of* $\Gamma(G^{pc}, \delta)$, *if and only if there is* $k \geq 0$ *such that* $\bar{m} = (kB^1, \ldots, kB^n)$.

Proof/

First suppose $k = 0$. Then $\bar{m} = \tilde{m} = (0, \ldots, 0)$. But clearly, $s_{\tilde{m}}^{\bar{m}}$ is an SPE for any δ since it $\tilde{m}$ is the only Nash equilibrium of the one-shot game.

Now suppose that $k > 0$. To see that $s_{\tilde{m}}^{\bar{m}}$ is also an SPE consider any $i \in N$ any $h \in \mathcal{H}$. This history could have evolved in one of two ways.

Suppose first that the game has not ended at any given t, that $h_t \in \mathcal{H}^{\alpha}$, and that the other agents have been playing kB^{-i} each round. If i makes the optimal defection from $m^i = kB^i$ then his payoff is δB^i. This is because he gets the benefit of free riding in round t if the game ends. If the game does not end at t then all agents report 0 in the next round, and the game ends with certainty in the subsequent round with no public good because of the non-stationary play. On the other hand, not defecting gets a payoff of $B^i - \frac{B^i C}{\sum_{j=1}^{n} B^j}$ since the k's cancel. But by construction:

$$\delta B^i = \left(1 - \frac{C}{\sum_{j=1}^{n} B^j}\right) B^j = B^i - \frac{B^i C}{\sum_{j=1}^{n} B^j}.$$

Thus, it is a best response for i to play kB^i since no additional benefit is gained by defecting.

Suppose on the other hand that at some $t \in \{1, 2, \ldots\}$, some agent j plays $\hat{m}^j \neq kB^j$. Then by lemma 2, it is a best response for all agents to abide by the trigger strategy and play $m = 0$ in all remaining rounds.

Finally, suppose there was another trigger strategy, $s_{\tilde{m}}^{\hat{m}}$, that was an SPE. But then for some agent $i \in N$, $\hat{m}^i > kB^i$. In this case:

$$\delta B^i = \left(1 - \frac{C}{\sum_{j=1}^{n} B^j}\right) B^i > B^i - \frac{\hat{m}^i C}{\sum_{j=1}^{n} \hat{m}^j},$$

and defecting has a higher expected payoff than abiding by the trigger strategy.

•

Thus we have:

Theorem 2. *Suppose* $C \leq \sum_{j=1}^{n} B^j$. *Then if* $\delta = 1 - \frac{C}{\sum_{j=1}^{n} B^j}$, $\{(B^1 - \frac{CB^1}{\sum_{j=1}^{n} B^j}, \ldots, B^n - \frac{CB^n}{\sum_{j=1}^{n} B^j}), (0, \ldots, 0)\}$ *are the only SPE payoffs of the PCT extension* $\Gamma(G^{pc}, \delta)$

Proof/

By lemma 7, $s_{\bar{m}}^{\bar{m}}$ is an SPE trigger strategies if and only if for some $k \geq 0$, $\bar{m} = (kB^1, \ldots, kB^n)$. Then by lemma 3,

$$\bar{h}^k \equiv \{h \in \mathcal{H} \mid \forall t \in \{1, 2, \ldots\}, (kB^1, \ldots, kB^n) \in_t \bar{h}^k\},$$

for all $k \geq 0$ are the only SPE histories. Then since for all $k > 0$,

$$E_1(\bar{m}, h_1) = (B^1 - \frac{CB^1}{\sum_j = 1^n B^j}, \ldots, B^n - \frac{CB^n}{\sum_{j=1}^{n} B^j}),$$

and for $k = 0$,

$$E_1(\bar{m}, h_1) = (0, \ldots 0)\},$$

these are the only SPE payoffs.

•

Thus, the mechanism we describe here has the property that if the mediator knows the sum of the benefits, he can choose δ such that the only equilibrium payoffs are sharing in proportion to the benefits, and the one-shot Nash equilibrium in which the project is not built. We can interpret this game as a bargaining problem. The disagreement point is $(0, \ldots, 0)$, the payoffs to the agents if the project is not built. The best possible situation for any given agent is for project to be built and he free rides. Therefore, the *Ideal Point* (in the sense of Kalai) is just $(B^1, \ldots, B^n)$. Notice that if for some $i \in N$, B^i increases, the net payoff to that agent,

$$B^i - \frac{B^i C}{\sum_{j=1}^n B^j},$$

also goes up. Proportional sharing therefore satisfies *Restricted Monotonicity.* In addition, sharing costs in proportion to benefits is clearly 1)Pareto optimal, 2) Symmetric when benefits are symmetric and, 3) Scale invariant. Thus, this solution is just the Kalai-Smorodinsky solution.

Finally, we show that regardless of δ, if the sum of the benefits is less than the cost, building the project can never be an equilibrium. This is important if the mediator makes a mistake, or for some reason, incorrectly estimates the total benefits. It means that a bad projects will never be built regardless of these errors.

Lemma 8. *Suppose* $C > \sum_{j=1}^n B^j$. *Then for all* $\delta > 0$, $s_{\tilde{m}}^{\tilde{m}}$ *is the only SPE trigger strategy of* $\Gamma(G^{pc}, \delta)$.

Proof/

Clearly, $s_{\tilde{m}}^{\tilde{m}}$ is an SPE since $\tilde{m}$ is the only Nash equilibrium of the one-shot game.

To see that there can be no other equilibrium note the following. Since $C > \sum_{j=1}^n B^j$, for all $\hat{m} \neq \tilde{m}$, it must be that for some agent $i \in N$

$$\frac{\hat{m}^i}{\sum_{j=1}^n \hat{m}^j} > \frac{B^i}{\sum_{j=1}^n B^j}.$$

Also, for all $i \in N$,

$$C \frac{B^i}{\sum_{j=1}^n B^j} > B^i$$

Then for all $\delta > 0$,

$$\delta B^i > 0 > B^i - \frac{B^i C}{\sum_{j=1}^n B^j} > B^i - \frac{m^i C}{\sum_{j=1}^n m^j}.$$

Thus, for all $\delta > 0$, defecting yields positive expected payoff, while abiding by any trigger strategy other than one enforcing $\tilde{m}$ yields a negative expected value.

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5. Self Enforcing Optimal Urban Planning

Consider the following one-shot game: $G^{up} \equiv < N, M, v >$. Here each agent $i \in N$, chooses between two moves: $M^i = \{CC, MB\}$ (Comply with the Code, or Maximize private Benefit). Let $\#CC : M \rightarrow \{0, \ldots, n\}$ be a function that gives the number of agents who agree to comply with the code in any given set of moves. Then for all $i \in N$, and all $m \in M$,

$$v^i(m) = \begin{cases} B^i\#CC(m) - C^i & \text{if } m^i = CC \\ B^i\#CC(m) & \text{if } m^i = MB. \end{cases}$$

We interpret B^i as the external benefit that agent i receives when any agent undertakes the socially beneficial action (complying with the code), and C^i as the private cost of undertaking this action himself. We assume that for all $i \in N, C^i > B^i$. This means that the only Nash equilibrium of the one-shot game is for each agent to maximize the private benefit of his property. In fact, this is a dominant strategy equilibrium.

This is a model of an urban planning game. If an agent complies with the code, he generates benefits for himself and his neighbors. However, the private cost of compliance is higher than the private benefit. To fix the idea, we can imagine a group of developers filing plans with the building commission. The random deadline aspect might be generated by uncertainty over when the building commission meets. The game described above is substantially different from the first two. The problem before was to divide the cost of a discrete level of public good. This game is directed toward assuring the provision of an efficient level of a positive externality, or the abatement of a negative externality. The attraction of thinking about this specifically as an urban planning game is that putting up a development is fundamentally a one-shot proposition.

Let us first consider the case of identical agents. Let

$$\delta^* = \frac{B \mid N \mid - C}{B(\mid N \mid - 1)}$$

$\tilde{m} \equiv (\tilde{m}^1, \ldots, \tilde{m}^n) = (MB, \ldots, MB)$, and $\bar{m} \equiv (\bar{m}^1, \ldots, \bar{m}^n) = (CC, \ldots, CC)$.

Lemma 9. *Suppose all agents in the game G^{up} are identical. Then for the PCT extension $\Gamma(G^{up}, \delta^*)$, $s^{\tilde{m}}_{\tilde{m}}$ and $s^{\bar{m}}_{\tilde{m}}$ are the only SPE.*

Proof/

Clearly, $s^{\tilde{m}}_{\tilde{m}}$ is an SPE for any δ since it $\tilde{m}$ is the only Nash equilibrium of the one-shot game.

To see that $s^{\bar{m}}_{\tilde{m}}$ is also an SPE consider any $i \in N$ any $h \in \mathcal{H}$. This history could have evolved in one of two ways.

Suppose first that the game has not ended at any given t, that the other agents have been playing $\bar{m}^{-i}$ each round. If i makes the optimal defection from $\bar{m}$ and tries to free ride, then his expected payoff is $\delta^*(\mid N \mid - 1)B$. On the other hand, not defecting gets a payoff of $B \mid N \mid - C$ with certainty. But by construction:

$$\delta^* B(\mid N \mid - 1) = \frac{B \mid N \mid - C}{B(\mid N \mid - 1)} B(\mid N \mid - 1) = B \mid N \mid - C.$$

Thus it is a best response for i to play CC since no additional benefit is gained by defecting.

Suppose on the other hand that at some $t \in \{1, 2, \ldots\}$, some agent j plays $\hat{m}^j \neq \bar{m}^j$. Then by lemma 2 it is a best response for all agents to abide by the trigger strategy and play FR in all remaining rounds.

Finally, suppose there was another trigger strategy $s^{\hat{m}}_{\tilde{m}}$ that was an SPE. But then $\#CC(\bar{m}) > \#CC(\hat{m})$ and $\delta^* < 1$ so

$$\delta^* B(\#CC(\hat{m}) - 1) = \delta^* B(\mid N \mid - 1) - \delta^* B(\mid N \mid - \#CC(\hat{m}))$$

$$= B \mid N \mid - C - \delta^* B(\mid N \mid - \#CC(\hat{m})) > \#CC(\hat{m})B - C.$$

Thus it would be optimal for every agent to defect from any other trigger strategy.

•

Thus we have:

Theorem 3. *Suppose that all the agents are identical. Then for the PCT extension* $\Gamma(G^{up}, \delta^*)$, $\{(B \mid N \mid - C, \ldots, B \mid N \mid - C), (0, \ldots 0)\}$ *are the only SPE payoffs.*

Proof/

By lemma 9, $s^{\bar{m}}_{\bar{m}}$ and $s^{\tilde{m}}_{\tilde{m}}$ are the only SPE trigger strategies. Then by lemma 3,

$$\bar{h} \equiv \{h \in \mathcal{H} \mid \forall t \in \{1, 2, \ldots\}, \bar{m} \in_t \bar{h}\},$$

and

$$\tilde{h} \equiv \{h \in \mathcal{H} \mid \forall t \in \{1, 2, \ldots\}, \tilde{m} \in_t \bar{h}\},$$

are the only SPE histories. Thus

$$\{(B \mid N \mid - C, \ldots, B \mid N \mid - C), (0, \ldots 0)\}$$

are the only SPE payoffs.

•

This result also holds if agents are almost identical, or if at least the ratios of costs to benefits increases sufficiently slowly. It is possible to prove a result similar to lemma 5 for this game. We will not do so here because the sufficiency condition is less easy to interpret. But it turns out that if we order the agents so that those with a lower index have a lower ratio of costs to benefits then if for all $i \in \{1, \ldots, n-1\}$,

$$1 \leq \left(\frac{C^i}{B^i} - \frac{C^{i+1}}{B^{i+1}}\right) + \frac{C^{i+1}}{B^{i+1}}$$

then for

$$\delta = \frac{B^n \mid N \mid - C^n}{B^n(\mid N \mid - 1)}$$

the only SPE trigger strategies have all agents abiding by the code or all agents maximizing private benefits. If the cost-benefit ratio increases faster than this, there may also be other SPE trigger strategies.

6. Bertrand Oligopoly

Next we describe the case of constant marginal cost Bertrand oligopolists facing a known demand curve. For simplicity, we will assume that demand is linear, but this is easily generalized. Let the demand be given by

$$Q = \alpha - \beta p,$$

and define $LPF : \Re^n_+ \rightarrow \{I \mid I \subseteq N\}$:

$$LPF(m) \equiv \{I \subseteq N \mid i \in I \text{ if and only if } \forall\, j \in N, m^i \leq m^j\}.$$

This correspondence gives the subset of Lowest Priced Firms. Now, consider the one-shot game $G^{bo} \equiv < N, M, v >$ where for all $i \in N$, $M^i \equiv \Re_+$, and for all $i \in N$ and all $m \in M$:

$$v^i(m) = \begin{cases} \frac{(\alpha-\beta m^i)(m^i-C^i)}{\mid LPF(m) \mid} & \text{if } i \in LPF(m) \\ 0 & \text{otherwise.} \end{cases}$$

Thus, all of the lowest priced firms share the demand equally. Other firms sell nothing. It is well known that the only Nash equilibrium in the case of identical firms is for each firm to price at cost. We restrict attention to this case. Let $\tilde{m} = (C, \ldots, C)$.

Lemma 10. *Suppose all firms are identical. Then for* $\delta = \frac{1}{\mid N \mid}$ $s^{\bar{m}}_{\tilde{m}}$ *is an SPE trigger strategy of* $\Gamma(G^{bo}, \delta)$, *if and only if for all* $i, j \in N$ $\bar{m}^i = \bar{m}^j$, *and* $\bar{m} \in [C, \frac{\alpha+\beta C}{2\beta}]$.

Proof/

First suppose that $\bar{m} = \tilde{m} = (C, \ldots, C)$. Then clearly, $s^{\bar{m}}_{\tilde{m}}$ is an SPE for any δ since it $\tilde{m}$ is the only Nash equilibrium of the one-shot game.

Now suppose that $\bar{m} \in (C, \frac{\alpha+\beta C}{2\beta}]$. To see that $s^{\bar{m}}_{\tilde{m}}$ is also an SPE consider any $i \in N$ and any $h \in \mathcal{H}$. This history could have evolved in one of two ways.

Suppose first that the game has not ended at any given t, and that the other agents have been playing $\bar{m}^{-i}$ each round. By hypothesis, the equilibrium price is below the monopoly price, $\frac{\alpha+\beta C}{2\beta}$. Thus, optimal defection for any $i \in N$ is to lower the price by ϵ and capture all the demand. His expected payoff in this case is

$$\delta(\alpha - \beta(\bar{m} - \epsilon))(\bar{m} - \epsilon - C).$$

This is because all agents revert to playing C if the game happens not to end in the round that i defects. On the other hand, by not defecting agent i gets a payoff of

$$\frac{(\alpha - \beta\bar{m})(\bar{m} - C)}{\mid N \mid}$$

with certainty. But by construction:

$$\delta(\alpha - \beta(\bar{m} - \epsilon))(\bar{m} - \epsilon - C) < \delta(\alpha - \beta\bar{m})(\bar{m} - C) = \frac{(\alpha - \beta\bar{m})(\bar{m} - C)}{\mid N \mid}.$$

Thus, it is a best response for i to play $\bar{m}$ since no additional benefit is gained by defecting.

Suppose on the other hand that at some $t \in \{1, 2, \ldots\}$, some agent j plays $\hat{m}^j \neq \bar{m}^j$. Then by lemma 2 it is a best response for all agents to abide by the trigger strategy and play $m^i = C$ in all remaining rounds.

Finally, suppose there is another trigger strategy, $s^{\hat{m}}_{m}$, that is an SPE but does not satisfy the hypothesis of the lemma. Note first that if $s^{\hat{m}}_{m}$ is an SPE, then for all $i, j \in N$, $\hat{m}^i = \hat{m}^j$. This is because any agent who names a price which is not the lowest receives no profit. Thus for all $\delta > 0$, the optimal defection would necessarily have a positive expected value. Therefore, no agent could name a price above the lowest price in equilibrium. Second, clearly $\hat{m} \geq C$. Otherwise defection would give agents positive profits instead of negative profits. Finally, suppose that $\hat{m} > \frac{\alpha+\beta C}{2\beta}$. In this case, it is optimal to defect to the monopoly price. This gives an expected profit of

$$\delta(\alpha - \frac{\alpha + \beta C}{2})(\frac{\alpha + \beta C}{2\beta} - C).$$

But for all $\bar{m}^i > \frac{\alpha+\beta C}{2\beta}$,

$$\delta(\alpha - \frac{\alpha+\beta C}{2})(\frac{\alpha+\beta C}{2\beta} - C) > \frac{(\alpha - \beta\bar{m}^i)(\bar{m}^i - C)}{\mid N \mid},$$

since it is easy to check that $m = \frac{\alpha+\beta C}{2\beta}$ is profit maximizing.

•

Thus we have:

Theorem 4. *Suppose that all the agents are identical. Then for the PCT extension* $\Gamma(G^{bo}, \frac{1}{\mid N \mid}, \{\frac{(\alpha-\beta m)(m-C)}{\mid N \mid}, \ldots, \frac{(\alpha-\beta m)(m-C)}{\mid N \mid} \mid m \in [C, \frac{\alpha+\beta C}{2\beta}]\}$, *are the only SPE payoffs.*

Proof/

By lemma 10, $s^{\bar{m}}_{\bar{m}}$ for $\bar{m} \in [C, \frac{\alpha+\beta C}{2\beta}]$ are the only SPE trigger strategies. Then by lemma 3,

$$\bar{h} \equiv \{h \in \mathcal{H} \mid \forall t \in \{1, 2, \ldots\}, \bar{m} \in_t \bar{h}\},$$

for $\bar{m} \in [C, \frac{\alpha+\beta C}{2\beta}]$ are the only SPE histories. Thus

$$\{\frac{(\alpha - \beta m)(m - C)}{\mid N \mid}, \ldots, \frac{(\alpha - \beta m)(m - C)}{\mid N \mid} \mid m \in [C, \frac{\alpha + \beta C}{2\beta}]\}$$

are the only SPE payoffs.

•

Thus, all the mediator needs to know to choose the appropriate δ is the number of firms in the in the market. Equal sharing of the monopoly profit is the unique Pareto dominant SPE equilibrium payoff.

It may be possible to sharpen this result by requiring that the defecting firm lower his price by at least a fixed ϵ (instead of the arbitrarily small ϵ here) in order to capture the market. This seems to reduce the equilibrium set to three elements:

the monopoly price, the competitive price, and ϵ above the competitive price. The problem is that these last two are also equilibria of the one-shot game. We therefore could not use the lemmata proved in the early sections since the hypothesis that there be only one equilibrium in the one-shot game is not met. Consequently, we do not pursue this further in the current paper.

7. Chain Store Paradox

In this section we depart from the mechanism design approach above and address the positive implications of probabilistic cheap talk. Our focus is on the chain store paradox offered by Selten (1978). However, it should become clear that the fundamental rationale can be applied to explain the much larger class of phenomena in which one party needs to make a commitment to an action that appears to be an irrational choice in a one-shot game.

A resolution of the chain store paradox has previously been suggested by Kreps, and Wilson (1982) and Milgrom, and Roberts (1982), based on a model of incomplete information on the entrant's part regarding the type of the incumbent chain store. This explanation may be criticized on the grounds that it places too much reliance on the existence of a "crazy" type of incumbent. It would be desirable to obtain an explanation for how a chain store could maintain its monopoly without abandoning the fundamental assumption of "sanity" which is inherent in virtually every other model of economic behavior. Such an explanation is given below.

The apparent paradox is as follows. Consider a game, denoted G^{cs} with two types of players. The chain store is an incumbent (I) located in k distinct markets. The potential entrants ($i = 1, \ldots, k$) must make their decisions sequentially. Assuming there is a one-to-one correspondence between entrants and markets, we shall us i to denote both markets and entrants. Entrant i's decision is based upon the observation of moves made by the entrants and incumbent in previous markets $j = 1, \ldots, i-1$. The extensive form of the game between entrant i and incumbent I in market i is given in Figure 1.

With $i = k = 1$, it is clear that the unique SPE of the game is for i to choose to enter the market (In) and I to choose to acquiesce (Acq). By induction, for arbitrary k, the unique SPE involves each i entering market i, and I acquiescing

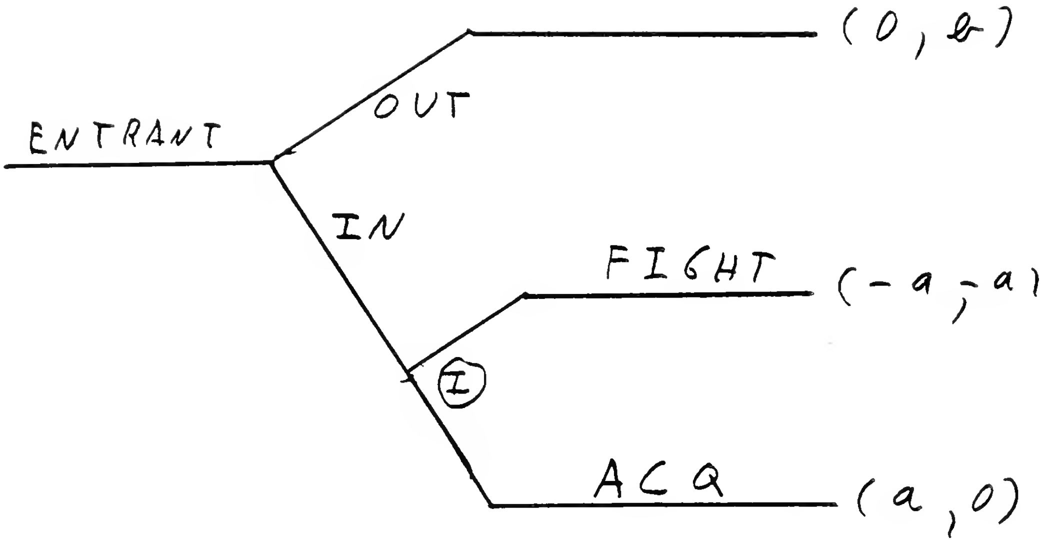

Figure 2

each time. Since it does not make sense to think about an infinity of markets, it is not possible to turn this into an infinitely repeated game and prove a folk theorem.

How do we rationalize the outcome in which I threatens to fight if there is entry and the entrant decides to stay out of the market? Consider again $i = k = 1$. Also, to keep the story simple assume there is a single consumer of the product and there is some probability each round that he complete the purchase of the good.[3] This is consistent with reality; prices are often set with the expectation that there will be time to revise the offering before the actual purchase takes place. Up until the consumer accepts the offer, the prices are cheap talk. The consumer is not

[3] An alternative way of looking at this problem is to assume that the consumer will decide to make a purchase at some point in the finite time interval $[0, T)$, but that the exact moment is not known. Thus agents know that a purchase will take place this week, (for example) but can change their offering prices up until the moment that the consumer actually happens to walk through to the door.

considered a strategic player in this problem.

Consider the following pair of strategies that give the outcome described above. The incumbent fights each round in which there is entry if he has never acquiesced in the past, otherwise he acquiesces if there is entry. The entrant stays out each round unless has entered in the past and the incumbent has acquiesce, in which case he enters in this and all subsequent rounds. To see that this is indeed an SPE, suppose the game has lasted until round t, and consider any history of play h_t. There are only two types of subgames for the entrant. If at some point in the history he has entered and the incumbent acquiesced, it is clearly a best response to follow the strategy and enter now and in the future. This gives the entrant his maximal payoff of a. On the other hand, if all previous entrances (if any) to the market have been fought, then staying out gives a payoff of zero, instead of a loss of $(-a)$. For the incumbent, we have to show that this strategy is payoff maximizing in both the subgame in which the entrant comes in and stays out for all h_t. If the incumbent has ever acquiesced to entrance before, then it best response to acquiesce in the future regardless of the entrant's move. This gives an expected payoff of zero instead of $-a$ On the other hand, suppose that the all previous entrances have been fought. Then if the entrant stays out then clearly fighting if there is entry is a best response. In this case, following the strategy results in the entrant always staying out in the future, and the incumbent receiving his maximal payoff b whenever the game happens to end. If the entrant comes into the market, and the incumbent follows his strategy, there is a probability of δ that the consumer will make the purchase and the incumbent will get a payoff of $-a$. There is a $(1-\delta)$ probability that he consumer will not make his purchase, the entrant will exit and stay out, and so the incumbent will get his monopoly profit of b when the game finally ends. Thus the expected payoff to the incumbent from following the strategy in this subgame is

$$\delta(-a) + (1-\delta)(b).$$

Not fighting gives a payoff of zero in round t and in all future rounds. Any attempt to fight in the future will serve only to lower profits more. Therefore, for small enough δ, the strategy is a subgame perfect response for the incumbent as well.

Thus, the uncertainly about when the purchase will take place gives the chain store game a probabilistic cheap talk flavor that can be exploited to explain the "paradox" without resorting to irrational players. It is probably also possible to get a similar result with a chain store model in which purchases are made each round, but there is uncertainty about when the last purchase will be made.

8. Conclusion

In conclusion, this paper may be viewed as taking an alternative approach to the problem of implementing social choice functions. The implementation paradigm is to propose a mechanism that has the allocation suggested by the a given social choice rule as its unique equilibrium. This is a very hard thing to accomplish in a completely satisfactory way. The various impossibility results, and the complexity of many of these mechanisms give testament to this. Our approach is to start with a very simple mechanism that can applied to any one-shot game. We concentrate on games in which there is a unique Pareto inefficient Nash equilibrium. This mechanism provides a "resolution" to these one-shot games in the sense that the set of subgame perfect equilibria are Pareto ranked with the Pareto superior solution being the one chosen by the social choice rule. While this is weaker than implementing a social choice rule, we argue that it is more than enough to assure that the desirable outcome is achieved. The problem is reduced to one of coordinating on the SPE which is unanimously strictly preferred by all agents. We give examples which show that if we are willing to be satisfied with this weakening of implementation, then it is possible to resolve the free rider problem, and the revelation problem in simple public goods economies, the externality problem, and the prisoners' dilemma problem of Bertrand oligopoly. These resolutions are possible without violating the fundamental constraints of one-shot simultaneous move games, or altering the private information structure. Moreover, the equilibrium set of the PCT extensions of the one-shot games always has a unique Pareto dominate SPE which is feasible, Pareto efficient, and individually rational.

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